The equations being assumed linearly independent, no row has each term zero, nor can any row be made to have each term zero by addition of multiples of other rows. If $\alpha_0^{(0)} = 0$ then we can move to the position of first column a column where the term in the first row is different from zero since not all the terms of any row are zero. Then by addition of multiples of this row all but the first term of the first column can be made zero. If $\alpha_1^{(1)} = 0$ then some column following has the term in the second row different from zero and it can be brought to occupy the position of second column. As before all the terms of this column below the second can be made zero by addition of multiples of the second row. Proceeding in this way a new matrix is obtained the first $(n + 1)$th order determinant of which will have all the terms below the principal diagonal equal to zero and each term of that diagonal different from zero. The determinant of the original matrix composed of those $n + 1$ columns which form the first $(n + 1)$th order determinant of the transformed matrix, is obviously not zero. Hence if the equations are linearly independent there will be at least one $(n + 1)$th order determinant different from zero, and hence if every $(n + 1)$th order determinant of the coefficients in (7) is zero the equations are linearly dependent.

In the case of linear dependence the equations which are dependent on others can be omitted and the general solution can be found from the linearly independent equations. This can be done as above by making arbitrary, with restriction as to convergence, all the u's except as many as there are equations and solving for these by the ordinary method.

PRECISE MEASUREMENTS WITH A STEEL TAPE OR WIRE.

By GEORGE R. DEAN, Rolla, Mo.

In the measurement of horizontal distances with a steel tape or wire in cases where all attainable accuracy is required, there will be considerable calculation avoided if the engineer has the means of knowing how to adjust the tension so that the elongation due to tension may balance the correction for sag or droop. Under ordinary conditions the correction for temperature is negligible, and when not negligible is easily applied and need not be discussed here.

We will derive a formula for the sag correction and the stretch separately. Then by equating these corrections, derive a formula for the tension in terms of the weight, cross-section and modulus of elasticity of the tape or wire.

Correction for Sag.—This is usually derived by the aid of the calculus in text-books on mechanics, but the calculus is unnecessary for this purpose, if we assume that the curve is the arc of a circle. The approximate formula derived in this way is the same as that obtained by integration when the curve is taken as a catenary or a parabola.

Let R = radius of circle,
l = length of tape or wire in inches,
α = number of radians in half the angle at center,
h = horizontal distance in inches,
δ = sag in inches.

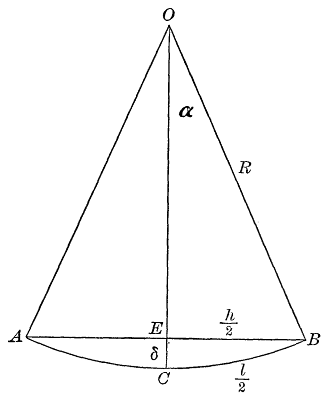

FIG. 1.

In Fig. 1,

$$OA = OB = OC = R,$$

$$\angle\ COB = \alpha, \quad \text{and} \quad EC = \delta.$$

$$AB = h, \quad ACB = l.$$

Then

$$l = 2R\alpha, \tag{1}$$

$$h = 2R \sin \alpha, \tag{2}$$

$$\delta = R(1 - \cos \alpha). \tag{3}$$

Since α is in practice a very small angle, we may write as approximations,

$$\sin \alpha = \alpha - \frac{\alpha^3}{6}, \quad \text{and} \quad \cos \alpha = 1 - \frac{\alpha^2}{2},$$

using the first two terms of the sine and cosine series. Then our equations (1), (2), (3), become

$$l = 2R\alpha, \tag{4}$$

$$h = 2R\left(\alpha - \frac{\alpha^3}{6}\right), \tag{5}$$

$$4\delta = 2R\alpha^2. \tag{6}$$

Dividing (6) by (4),

$$\alpha = \frac{4\delta}{l}. \tag{7}$$

Dividing (5) by (4),

$$1 - \frac{\alpha^2}{6} = \frac{h}{l}. \tag{8}$$

From (8),

$$h = l - \frac{l\alpha^2}{6}.$$

Substituting value of α from (7)

$$h = l - \frac{8\delta^2}{3l}. \tag{9}$$

Then the correction for sag is $8\delta^2/3l$.

Correction for Tension.—Let T be the pull at each end, w the weight of tape or wire per unit length, a the area of cross-section in square inches, E the modulus of elasticity of the material, θ the angle between the direction of the pull and the horizontal. The smaller the sag the more nearly will CF be equal to CE (Fig. 2).

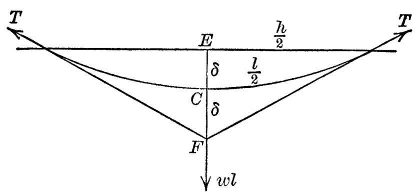

Fig. 2.

The forces T, T and wl are in equilibrium. The vertical component of each pull is $T \sin \theta$. Then we have

$$2T \sin \theta = wl. \tag{10}$$

Now

$$\sin \theta = \frac{2\delta}{\frac{2}{l}} = \frac{4\delta}{l}, \text{ approximately.}$$

Then

$$2T\left(\frac{4\delta}{l}\right) = wl,$$

or

$$T = \frac{wl^2}{8\delta}. \tag{11}$$

If x denote the elongation,

$$x : l = \frac{T}{a} : E,$$

or

$$x = \frac{lT}{aE}. \tag{12}$$

Equating this to the correction for sag,

$$\frac{lT}{aE} = \frac{8\delta^2}{3l}. \tag{13}$$

From (11),

$$\delta = \frac{wl^2}{8T}.$$

Substituting in (13),

$$\frac{lT}{aE} = \frac{8w^2l^4}{64T^2 \cdot 3l} = \frac{8w^2l^4}{192T^2l}.$$

Solving for T,

$$T^3 = \frac{aEw^2l^2}{24}, \tag{14}$$

or

$$T = \sqrt[3]{\frac{aEw^2l^2}{24}},$$

and since $wl = W$ the weight of tape,

$$T = \sqrt[3]{\frac{aEW^2}{24}}. \tag{15}$$

Numerical Example.—If the weight of tape is 5 pounds, the modulus of elasticity 30,000,000 pounds per square inch, and the area of cross-section .015 square inch,

$$T = \sqrt[3]{\frac{.015 \times 30{,}000{,}000 \times 25}{24}} = 77.5 \text{ pounds.}$$

and

$$\delta = \frac{Wl}{8T} = \frac{5 \times 5}{8 \times 77.5 \times .015 \times 0.3} = \frac{25}{77.5 \times .036} = 8.9 \text{ inches.}$$

Here

$$l = \frac{5}{.015 \times 0.3} = 1{,}111.1 \text{ inches} = 92.5 \text{ feet.}$$

BOOK REVIEWS.

W. H. BUSSEY, Chairman of the Committee.

Practical Geometry and Graphics. A text book for students in technical and trade schools, evening classes, and for engineers, artisans, draughtsmen, architects, etc. By E. L. BATES and F. CHARLESWORTH. Van Nostrand, New York, 1912. viii + 621 pages. \$2.00.

Practical Mathematics. By E. L. BATES and F. CHARLESWORTH. Van Nostrand, New York, 1912. viii + 513 pages. \$1.50.

These two texts, which have been written in part to prepare for the examinations of the London Board of Education, are essentially what they claim to be, that is for practical men. In fact they almost take the form of an engineering pocket book. The ground covered is enormous, the proofs whenever given are brief and frequently of an experimental nature, such as would appeal to good